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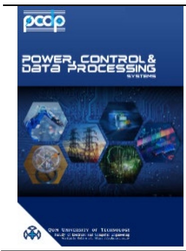

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Reliability-Oriented Optimal Placement of Thyristor-Controlled Phase-Shifting Transformers under Operational Uncertainty

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ABSTRACT

The Thyristor-controlled phase-shifting transformer (TCPST), a member of the flexible AC transmission systems (FACTS) family, offers an effective solution for mitigating transmission congestion while enhancing power system security and reliability. This paper introduces an innovative framework for determining the optimal location of TCPST to improve power system reliability under uncertainty. The proposed method accounts for power system uncertainties, including the availability of generation units and network components, modeled using forced outage rates (FORs) through scenario generation based on Monte Carlo simulation (MCS). To address the non-convex nature of the resulting mixed-integer non-linear programming problem, a linear approximation of power flow equations is employed, transforming the problem into a mixed-integer linear programming (MILP) formulation. The objective is to minimize load curtailment by identifying the optimal TCPST locations across different scenarios. The results are then ranked based on their contribution to reliability improvement, using the Probability of Load Curtailment (PLC) index. Extensive numerical studies on a modified reliability test system (RTS) demonstrate the efficacy of the proposed approach. Results show a 17.14% reduction in the annual Expected Energy Not Supplied (EENS) (from 646,640 MWh/year to 535,805 MWh/year) and a 25.29% improvement in the PLC index (from 0.344 to 0.257). These findings validate the effectiveness of the proposed stochastic optimization framework in enhancing power system reliability under uncertain conditions.

Keywords: Reliability; Uncertainty; Phase shifting transformer; Interrupted Energy Assessment Rate; Expected Energy Not Supplied.

1 Introduction

1.1 Motivation

Since the establishment of the first electric power networks between 1885 and 1895, power systems remained largely unchanged for many years [1]. In recent decades, however, developments in power system operation and planning have led to notable changes. At present, more than 9,200 power grids worldwide supply approximately one million megawatts of electricity. In a deregulated electric environment, the growth in power demand, together with competition among generating companies to increase their production share, has resulted in congestion in transmission systems [2]. As power systems move toward more optimized operation, it becomes harder to maintain flexibility and reliability [3, 4]. This is while the transmission congestion may lead to the tripping of overloaded lines, resulting in cascading events and blackouts. These widespread outages jeopardize the stability and reliability of modern power systems. Network reinforcement and expansion are two possible remedies to relieve this concern [5]. However, building new transmission lines is an expensive and time-consuming solution, which is restricted due to several factors such as right of way and environmental issues. As one of the most efficient strategies for enhancing the network's capabilities in the highest margins, FACTS devices propose a good alternative to avoid transmission line reinforcement or expansion schemes [6-8]. This observation emanates from their remarkable ability to dispatch the power flow patterns to avoid transmission overloading. The supportive effect of FACTS devices in power flow control through the transmission lines returns to voltage magnitude and phase angle adjustments at installation buses, altering the transmission line impedance through a series of inserted virtual reactance and supplying or absorbing reactive power [6, 9]. It should be noted that the installation location of FACTS devices and their rated sizing are two indispensable factors in guaranteeing the envisioned improvements [10, 11]. In this way, their location and size should be determined so appropriately.

The primary objective of this study is to determine the optimal placement of TCPST in power systems under uncertainty, aiming to minimize load curtailment and enhance system reliability. This is achieved through a stochastic optimization framework incorporating MCS, linearized power flow modeling, and the newly introduced PLCI index. Section 1.3 presents these objectives in more detail and explains how to cover the topic gaps.

1.2 Literature Review

Extensive research has been conducted on FACTS technologies' optimal location and sizing, employing diverse objective functions and a wide range of optimization algorithms to address various operational and economic challenges in power systems.

A valuable reference thoroughly analyzes the impact of FACTS devices on power quality and methods of optimal placement and settings of FACTS devices [12]. In work [13], the authors aim to improve the performance of power distribution systems by proposing a Mixed-Integer Linear Programming (MILP) model for the optimal placement, sizing, and operation of D-STATCOMs. The model incorporates multi-period optimization to account for daily variations in active and reactive power demand, ensuring dynamic reactive power support. Implemented in AMPL and solved with CPLEX, the approach achieves up to 23% loss reduction and 15% economic benefit, while enhancing voltage stability across 33-, 69-, and 136-bus test systems, demonstrating its effectiveness and scalability. In reference [14], a methodology is presented for the optimal placement of TCPST in power systems to enhance static stability reserves. The proposed approach utilizes a Genetic Algorithm (GA)-based optimization model to determine the optimal location for TCPST installation and identify the appropriate tap setting. Heshmatullah Nourizadeh and colleagues. In reference [15] , studies have been carried out on the optimal location of TCPSTs in a multi-year generation and transmission development planning problem in a pool-based market. It is worth mentioning that in this study, there is an objective function with conflicting goals: maximizing social welfare and minimizing investment costs. Another study uses a simple yet effective method for the optimal placement of FACTS shunts devices, which addresses critical stability challenges in modern power systems. This approach ensures system stability improvement during transient events and maintains system reliability [16]. In [17], a multi objective optimization methodology has been utilized to find the optimal allocation of TCPST to minimize the total fuel cost, transmission losses, and maximize system load ability. A mixed integer programming method in [18], is used to find the optimal placement of TCPST for maximizing system load ability. The authors' objective in [19], is to present an intelligent approach for optimizing the settings of TCPST to control unscheduled power flows in power systems. The study emphasizes the importance of coordinating TCPST settings to maximize their effectiveness in flow control. The authors aim in work [20], to optimize the size and placement of multiple FACTS devices to minimize fuel costs and power losses. They address the risk of local minimum entrapment using an Enhanced Tuna Swarm Optimization (ETSO) algorithm, improved with high and low-velocity ratios from the Marine Predator Algorithm (MPA). The IEEE 30- node, equipped with Static VAR Compensator (SVC), Thyristor Controlled Series Compensator (TCSC), Thyristor Control Phase Shifter (TCPS), and two wind turbines, serves as the case study. Optimal placement and rating of TCSC and SVC for congestion management in modern power systems by authors in reference [21], are analyzed. Also, the optimal location of FACTS devices for the enhancement of modern power systems loadability using continuation power flow by [22], has been studied. In addition, improved grid stability, reduced losses, and better power flow control by FACTS technologies in reference [23], have been studied. The study provides multi-period generation-transmission expansion planning with a TCPST allocation [24]. Also, solving the optimal distribution of stochastic reactive power by applying Beluga whale adaptive optimization considering uncertainty on the generation and demand side has been investigated by Mohamed Ebeed et al. in reference [25].Finally, reference [26], provides a probabilistic method to determine the optimal placement of TCPST to enhance modern power systems' transient stability. In [27], the authors proposed an enhanced metaheuristic algorithm, namely the Quick Crisscross Sine Cosine Algorithm (QCSCA), to address the OPF problem in power systems integrated with renewable energy sources and FACTS devices. The primary objective of the study was to minimize generation cost, power losses, and overall operating cost under both fixed and dynamic loading conditions. The novelty of the work lies in improving the conventional Sine Cosine Algorithm through adaptive parameter control, a Crisscross selection mechanism, and a Quick Move strategy, which collectively enhance convergence performance and prevent entrapment in local optima. In [28] the authors addressed the problem of overloaded power networks and associated power quality issues by optimally allocating FACTS devices in distribution systems. The main objective of the study was to formulate and solve a multi-objective optimization problem aimed at minimizing power loss, voltage deviation, and operational cost. To achieve this, a novel hybrid Genetic Algorithm–Improved Particle Swarm Optimization (GA-IPSO) method was proposed for the optimal sizing and placement of STATCOM, TCSC, and UPFC devices. The novelty of the work lies in the development of the GA-IPSO hybrid approach and its application to both separate and combined optimization of multiple FACTS devices, demonstrating improved technical and economic performance on the IEEE 33-bus radial distribution system. In [29], the authors investigated the internal fault current distribution characteristics of a TCPST under typical fault conditions. The primary objective of the study was to model the electromagnetic transient behavior of TCPST and analyze fault current levels during internal and external grounding faults. The novelty of the work lies in combining circuit transient equations with electromagnetic transient simulations to accurately characterize both transition and steady-state fault processes, as well as proposing and validating two fault current suppression strategies gear shifting and trigger pulse optimization to mitigate overvoltage and excessive fault currents. In [30], the authors focused on the optimal siting and sizing of TCPSTs in power systems with high renewable energy penetration. The main objective of the study was to develop a comprehensive planning framework that enhances economic efficiency, operational security, and renewable energy accommodation simultaneously. The novelty of the work lies in establishing a multi-dimensional evaluation indicator system and integrating Analytic Hierarchy Process (AHP) and Grey Relational Analysis (GRA) to determine indicator weights and construct a unified scoring mechanism. Based on this framework, an optimization model was formulated to identify the optimal location and capacity of TCPSTs, achieving a balanced trade-off among technical and economic objectives. In [31], the authors presented a comprehensive review of the role of FACTS controllers in deregulated power systems, particularly focusing on load frequency control, voltage stability, and congestion management. The main objective of the study was to systematically analyze the technical contributions of different types of FACTS devices (series, shunt, and combined controllers) and to compare the optimization techniques—ranging from classical and analytical methods to hybrid and meta-heuristic approaches used to enhance their performance. The novelty of this work lies in providing a structured and time-based comparative assessment of various generations of FACTS devices and associated optimization strategies, highlighting the growing dominance of third-generation controllers and hybrid/meta-heuristic techniques, while also emphasizing the economic trade-offs involved in their practical deployment.

1.3 Research Gap and Innovations

- Regardless of the extensive research in optimal FACTS device placement, the majority of previous research has relied on deterministic models with static system topologies, neglecting the uncertainties present in actual power systems like unexpected transmission lines and generator outages. Such research typically ignores the probabilistic aspect of power system operation, which can substantially impact the efficacy of FACTS device placement strategies in real-world situations.
- Unlike these classical methods, this work fills this pivotal research gap by creating a new and realistic framework for TCPST placement. Contrary to existing deterministic methods with assumed rigid system topologies, our framework considers operational availability uncertainty through the use of Monte Carlo simulation. This allows for the evaluation of multiple outages occurring concurrently, resulting in more accurate reliability estimations. Additionally, linearizing AC power flow equations results in converting the original non-linear problem to a Mixed Integer Linear Programming (MILP) model, lowering computational effort significantly but holding reasonable accuracy. We also propose a new PLCI index for ranking possible lines for TCPST installation based on their capability to enhance reliability.
- The main contributions of this research are as follows:

- Introducing a stochastic TCPST placement strategy taking into account generators' and transmission lines' forced outage rates (FORs).
- Applying a Monte Carlo simulation-based scenario generation strategy to capture topological and operational uncertainties.
- Reformulating the TCPST placement problem as a MILP to ensure scalable and efficient computation.
- Proposing the PLCI index for TCPST installation sequencing with probabilistic reliability impact.
- Validating the methodology through rigorous simulations on an enhanced reliability test system (RTS), which mirrors severely high increases in system reliability in conditions of uncertainty.

1.4 Paper structure

Section 2 discusses the approximation techniques applied to power flow equations. Section 3 addresses TCPST circuits and mathematical modeling in power flow studies. Section 4 launches the proposed methodology for TCPST placement incorporating reliability enhancement indices. Section 5 examines the simulation results of the proposed method and provides suitable discussions. Finally, Section 6 deals with conclusions, limitations, and future work.

2 Approximation of AC Power Flow

Fig. 1 presents a usual transmission line represented by its lumped-equivalent electrical circuit model between two buses. [32]. The mathematical statement of power flows from two buses can be represented by Eqs. (1) and (2) [33].

$$P_{i1,i2} = V_{i1}^2 G_{i1,i2} - V_{i1}V_{i2}[G_{i1,i2}\cos(\delta_{i1}-\delta_{i2}) + B_{i1,i2}\sin(\delta_{i1}-\delta_{i2})] \quad (1)$$

$$Q_{i1,i2} = -V_{i1}^2 (B_{i1,i2} + B_{sh}) - V_{i1}V_{i2}[G_{i1,i2}\sin(\delta_{i1}-\delta_{i2}) - B_{i1,i2}\cos(\delta_{i1}-\delta i_{b2})] \quad (2)$$

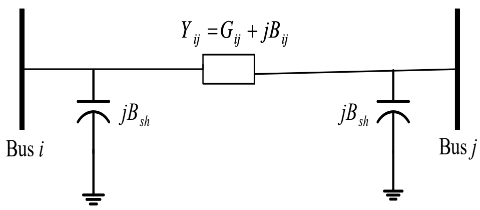

Fig. 1. π -equivalent circuit model of a transmission line.

To enhance the computational efficiency of the problem and for the sake of convexification, linear representation of (1)-(2) would result in a more tractable approach. As can be seen, these equations include trigonometric functions and product of continuous variables and are non-linear [34].

$$V_{i1}^2 = 2V_{i1} - 1 \quad (3)$$

$$V_{i1}V_{i2}\cos(\delta_{i1}-\delta_{i2}) = V_{i1} + V_{i2} + \gamma_{i1,i2} \quad (4)$$

$$V_{i1}V_{i2}\sin(\delta_{i1}-\delta_{i2}) = (\delta_{i1}-\delta_{i2}) \quad (5)$$

$$P_{i1,i2} = G_{i1,i2}(2V_{i1}-1) - G_{i1,i2}(V_{i1}+V_{i2}+\gamma_{i1,i2}-2) - B_{i1,i2}(\delta_{i1}-\delta_{i2}) \quad (6)$$

$$Q_{i1,i2} = -(B_{i1,i2}+B_{sh})(2V_{i1}-1) - G_{i1,i2}(\delta_{i1}-\delta_{i2}) + B_{i1,i2}(V_{i1}+V_{i2}+\gamma_{i1,i2}-2) \quad (7)$$

3 TCPST Modeling

Through the voltage phase angles regulation, the thyristor-controlled TCPST provides suitable controllability on power flow management. According to Fig. 2, a typical TCPST consists of two transformers. The first transformer, called an excitation transformer, is connected in parallel at the connection point. The second one is named the boosting transformer, which is incited by the phase-to-phase voltage of the former one. Technically, this transformer injects a series voltage with the transmission line perpendicular to the phase to neutral voltage [35]. The magnitude of the injected voltage is controlled by a converter as shown in Fig. 2.
Table.1, introduces the parameters used in TCPST modeling. The PST angle domain is assumed to be in the range of $[-15°, +15°]$. Also, the rating of PST is assumed to be equal with the rating of the line which PST is located in it.

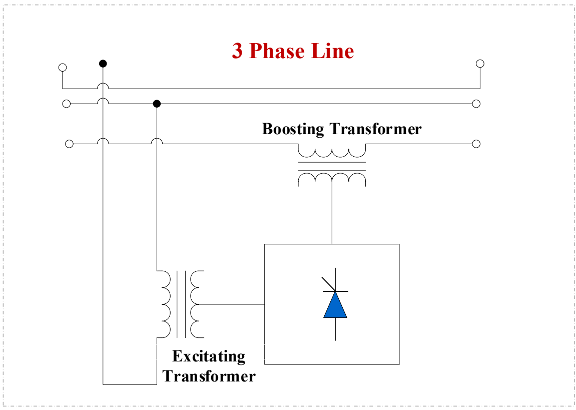

Fig. 2. A typical TCPST configuration.

Table.1. TCPST related parameters.

PST Location	L27	L18	L19	L22	L21	L28
From Bus	15	11	11	13	12	16
To bus	24	13	14	23	23	17
Rating of PST (MVA)	500	500	500	500	250	500

The power flow constraints of a branch consisting of TCPST at bus b1, are as follows [32, 36]:

$$P_{i1,i2} = V_{i1}^2 G_{i1,i2} - V_{i1}V_{i2}[G_{i1,i2}\cos(\delta_{i1}-\delta_{i2}+\varphi_{i1,i2}) + B_{i1,i2}\sin(\delta_{i1}-\delta_{i2}+\varphi_{i1,i2})] \quad (8)$$

$$P_{i1,i2} = V_{i1}^2 G_{i1,i2} - V_{i1}V_{i2}[G_{i1,i2}\cos(\delta_{i1}-\delta_{i2}+\varphi_{i1,i2}) + B_{i1,i2}\sin(\delta_{i1}-\delta_{i2}+\varphi_{i1,i2})] \quad (9)$$

$$P_{i1,i2} = V_{i2}^2 G_{i1,i2} - V_{i1}V_{i2}[G_{i1,i2}\cos(\delta_{i1}-\delta_{i2}+\varphi_{i1,i2}) - B_{i1,i2}\sin(\delta_{i1}-\delta_{i2}+\varphi_{i1,i2})] \quad (10)$$

$$Q_{i1,i2} = -V_{i1}^2(B_{i1,i2} + B_{sh}) - V_{i1}V_{i2}[G_{i1,i2}\sin(\delta_{i1} - \delta_{i2} + \varphi_{i1,i2}) - B_{i1,i2}\cos(\delta_{i1} - \delta_{i2} + \varphi_{i1,i2})] \quad (11)$$

$$Q_{i1,i2} = -V_{i2}^2(B_{i1,i2} + B_{sh}) + V_{i1}V_{i2}[G_{i1,i2}\sin(\delta_{i1} - \delta_{i2} + \varphi_{i1,i2}) + B_{i1,i2}\cos(\delta_{i1} - \delta_{i2} + \varphi_{i1,i2})] \quad (12)$$

It should be noted that, these equations have been also approximated based on a similar approach described earlier in Section 2.

4 Proposed TCPST Placement Methodology

This section proposes a stochastic framework for optimal TCPST placement, considering uncertainties of the power system components and the unavailability of generators and transmission facilities. The main skeleton of the optimal placement procedure is as follows. The MCS method is used to model the uncertainties associated with the outages of generators and transmission lines, considering them FOR values. Subsequently, the MCS approach determines the availability status of generators, transmission lines, and transformers in each scenario. To this end, the MCS approach is repeated in several executions. In each distinct run, the optimal TCPST placement is conducted for random system configurations with the objective of load shedding minimization. As the final step, the obtained results are ranked considering the least attainable PLC index.

4.1 Scenario Generation

In the scenario generation approach, a random value is determined between [0,1] based on a uniform probability distribution function [37] . This generated random value is utilized to assess each component's availability. To this end, the produced value is compared with the corresponding FOR value. If the generated random value exceeds the FOR value, then the corresponding component is recognized to be healthy. In contradiction, the equipment will be unavailable if the comparison results in a greater FOR value than the random one.

4.2 Optimal Mathematical Framework for TCPST Placement

As the initial step, the main objective function deployed in optimal locating TCPSTs is formulated comprehensively. The established formulation lies within the category of MILP problems, which have been tackled by utilizing the GAMS/CPLEX solver. Although the main option in handling the security constraint violation is either the FACTS adjustments or dispatches of generation units, by failing these actions, the load curtailment approach is the final remedy. Therefore, the objective function of our study is as follows:

$$Min \sum_{i1=1}^{N_i} \omega_{i1}.LC_{i1,s}^P \quad (13)$$

The objective function seeks for the minimum load curtailment considering the value of load curtailment in different buses. Variable $\omega_{i1}$is deployed to aggregate the system load curtailment in Eq. (13) using. This variable signifies the bus weighting factor. Different load shedding strategies would be considered by dedicating a different ω_{i1}value at each load bus. In this regard, the priority order policy [38], described in next sections, has been applied for effective load curtailment. Eqs. (14) and (15) are considered for assuring each bus's active and reactive power balance. The variables (P_{i1}^D) and (Q_{i1}^D) introduce the active and reactive power demands on bus b1, respectively. The modeling also defines active and reactive load curtailment on bus i using variables ($LC_{i1,s}^A$) and ($LC_{i1,s}^R$), respectively. (V) and (δ) represent the voltage magnitude and phase angle, respectively. Also, ($\gamma_{i1,i2}$) shows the piecewise linear approximation of ($\cos(\delta_i - \delta_j)$). Eqs. (16) and (17) cap the maximum active and reactive power curtailment at each bus in values smaller than the loading active and reactive powers. Eq. (18) is also included to preserve a constant power factor condition before and after the curtailment process. Eq. (19) restricts the flow of each line in the range of thermal loading capacity. Equations (20) and (21) show the minimum and maximum active/reactive power injection by the generating unit. Equation (22) also shows the voltage limitation [39, 40].

Eqs. (23) and (24) also denote the maximum and minimum values of TCPST angles and the limits for TCPST installation units.

$$P_{i1,s}^G + LC_{i1,s}^A - P_{i1,s}^D = \sum_{i2=1}^{N_i}\Big(G_{i1,i2}(2V_{i1} - 1) - G_{i1,i2}\big(V_{i1} + V_{i2} + \gamma_{i1,i2} - 2\big) - B_{i1,i2}\big(\delta_{i1} - \delta_{i2} + \varphi_{i1,i2}\big)\Big) \quad (14)$$

$$Q_{i1,s}^G + LC_{i1,s}^R - Q_{i1,s}^D = \sum_{i2=1}^{N_i}\Big(-(B_{i1,i2} + B_{sh})(2V_{i1} - 1) - G_{i1,i2}\big(\delta_{i1} - \delta_{i2} + \varphi_{i1,i2}\big) + B_{i1,i2}\big(V_{i1} + V_{i2} + \gamma_{i1,i2} - 2\big)\Big) \quad (15)$$

$$0 \le LC_{i1,s}^A \le P_{i1,s}^D \quad i1 = 1,2,\dots,N_i \quad (16)$$

$$0 \le LC_{i1,s}^R \le Q_{i1,s}^D \quad i1 = 1,2,\dots,N_i \quad (17)$$

$$\frac{LC_{i1,s}^A}{P_{i1,s}^D} = \frac{LC_{i1,s}^R}{Q_{i1,s}^D} \quad i1 = 1,2,\dots,N_i \quad (18)$$

$$|S_{l,s}| \le \overline{S_l} \quad l = 1,2,\dots,N_{br} \quad (19)$$

$$\underline{P_{i1}^G} \le P_{i1,s}^G \le \overline{P_{i1}^G} \quad i1 = 1,2,\dots,N_i \quad (20)$$

$$\underline{Q_{i1}^G} \le Q_{i1,s}^G \le \overline{Q_{i1}^G} \quad i1 = 1,2,\dots,N_i \quad (21)$$

$$\underline{V_{i1}} \le V_{i1,s} \le \overline{V_{i1}} \quad i1 = 1,2,\dots,N_i \quad (22)$$

$$-u_{i1,i2,s}\varphi_{ij} \le \varphi_{i1,i2,s} \le u_{i1,i2,s}\overline{\varphi_{i1,i2}} \quad (23)$$

$$\sum u_{i1,i2,s} = N_{PST} \quad (24)$$

Fig. 3 illustrates the flowchart of the proposed TCPST optimal placement method. At the initial step, the operating status of each component is assigned through the MCS approach. Then, in each MCS execution, the presented optimization problem is evaluated thoroughly, excluding the presence of TCPST. Then, the optimal location for TCPST placement is determined for scenarios with nonzero load curtailment. For each of the selected lines for TCPST placement, the Probability of load curtailment Improvement PLCI index is calculated based on the following relation:

$$\mathrm{PLCI}_l = \frac{\text{number of scenarios in which the PST placement in Line l totaly alleviates the load curtailment}}{\text{All of the investigated scenarios}} \tag{23}$$

Finally, the obtained locations according to the values of PLCI are sorted. The higher value for PLCI corresponds to a better TCPST location.

$$\mathrm{PLC}_l = \frac{\text{number of scenarios in which a part of customer loads is curtailed}}{\text{All of the investigated scenarios}} \tag{24}$$

$$\mathrm{EENS}_l = \frac{\textit{The} \text{ sum of not supplied load (MWh) in all senarios}}{\text{All of the investigated scenarios}} \tag{25}$$

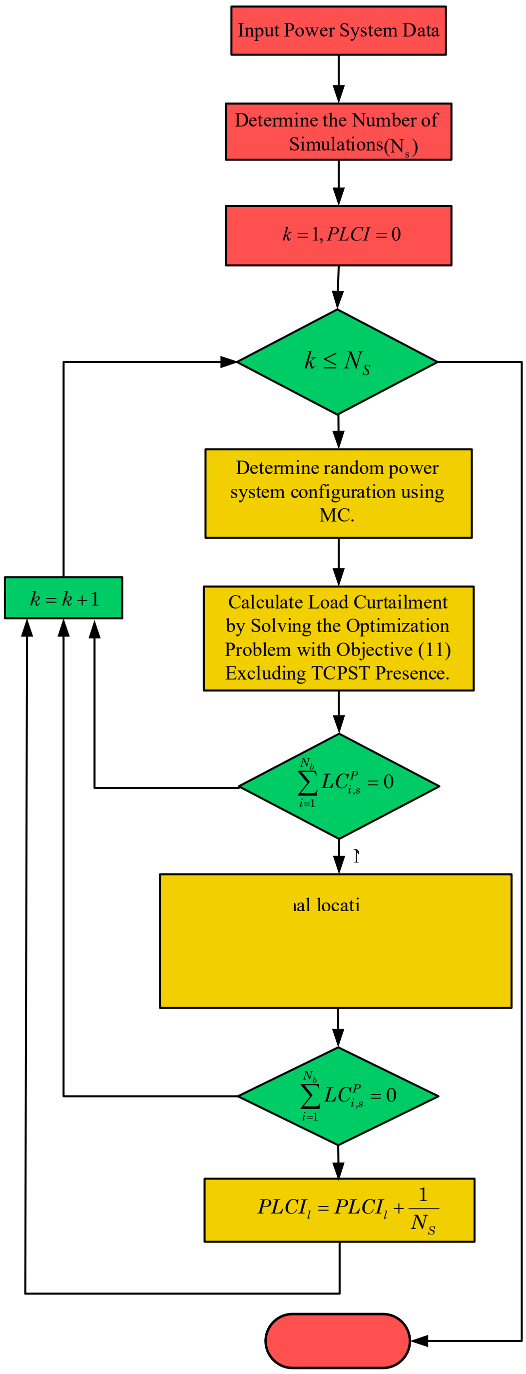

Fig. 3. Flowchart of the proposed TCPST optimal placement method.

4.2.1 Priority Order Policy

The Interrupted Energy Assessment Rate (IEAR), expressed in "$/kWh", is a reliability worth index commonly used in power system reliability analysis. It provides a monetary value for the expected cost of energy not supplied due to power interruptions. The highest IEAR has the highest priority, while the lowest IEAR has the lowest priority. If a major contingency occurs in the power system that calls for load reductions, the lowest priority delivery point is curtailed first. IEAR helps utilities and decision-makers evaluate and compare the cost-effectiveness of reliability improvement measures This policy will minimize customer interruption costs due to load curtailments. The priority order in IEEE-RTS follows Table.2 [38].

It should be noted that the proposed model relies on a linearized DC power flow approximation. While this approach significantly improves computational tractability, it neglects reactive power effects and voltage magnitude variations. Under heavily stressed or voltage-constrained conditions, AC nonlinear power flow models may provide more accurate representations. Therefore, the proposed framework is best suited for transmission-level reliability assessment where angle differences are small and voltage deviations are limited.

5 Numerical Analysis

5.1 Test System Specifications

Different numerical studies have been conducted to assess the performance of the proposed approach. For this purpose, the modified RTS system [41], depicted in Fig. 4, has been considered the test case network. To adequately adjust the system to the TCPST placement, some modifications, including a half reduction of the line's rating between buses 12 and 23, are included. As well, two 400 MW generating units are supplemented to Bus 23.

Table 2. IEEE-RTS data in simulations.

Delivery Point	IEAR ($/kWh)	Priority Order
BUS 1	8.9815	3
BUS 2	7.3606	5
BUS 3	5.8990	11
BUS 4	9.5992	1
BUS 5	9.2323	2
BUS 6	6.5238	9
BUS 7	7.0291	8
BUS 8	7.7742	4
BUS 9	3.6623	17
BUS 10	5.1940	14
BUS 13	7.2813	6
BUS 14	4.3717	16
BUS 15	5.9744	10
BUS 16	7.2305	7
BUS 18	5.6149	13
BUS 19	4.5430	15
BUS 20	5.6836	12

These generation units have reliability characteristics similar to the original system's primary 400 MW units. Thus, the total generation capacity is increased to 4205 MW. Likewise, the loading at buses 19 and 20 has been augmented by 400 MW. Consequently, the annual peak load is raised to 3650 MW. For the TCPST rating, it is assumed to be equal to the rating of the line that is selected to be installed.

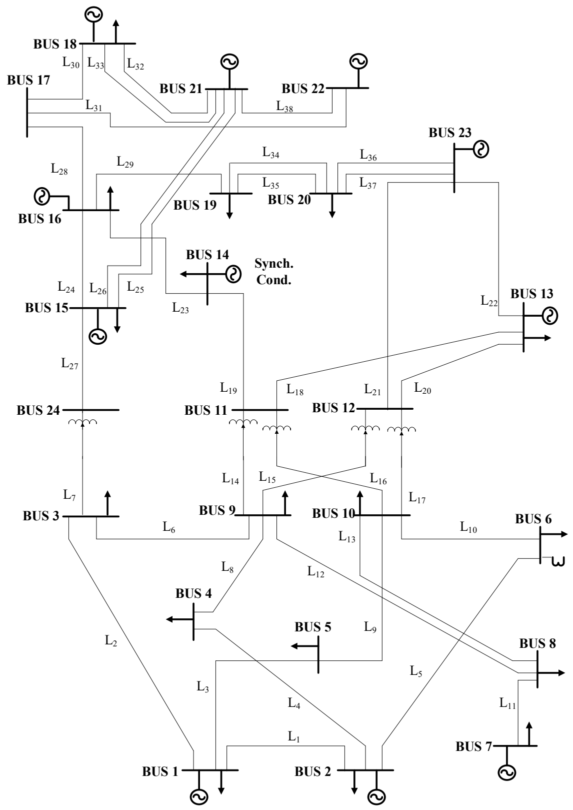

Fig. 4. Single-line diagram of IEEE RTS 24-bus.

5.2 AC Power Flow Approximation Validation

A numerical test case has been conducted on the RTS system to validate the approximation technique's accuracy. This system has been studied based on both the approximated method and the exact approach. The voltage magnitudes and phase angles acquired for all buses in the investigated system are depicted in Figs. 5 and 6, respectively. The linear approximation utilized by the given method presents minor errors in voltage magnitudes (<0.2%) and phase angles (<3.7%), as tabulated in Table.3. The given approximations may slightly reduce accuracy at extreme loading levels but at a reasonable compromise between solution accuracy and computational practicability, especially for the high number of scenarios under consideration within the stochastic method.

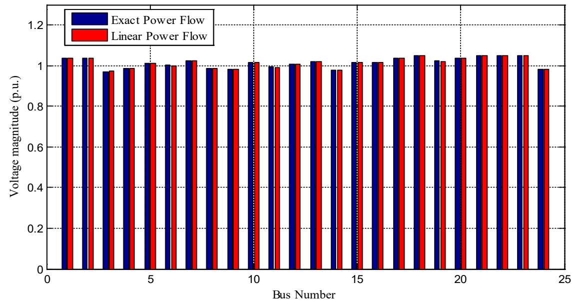

Fig. 5. Voltage magnitude validation through the approximated approach.

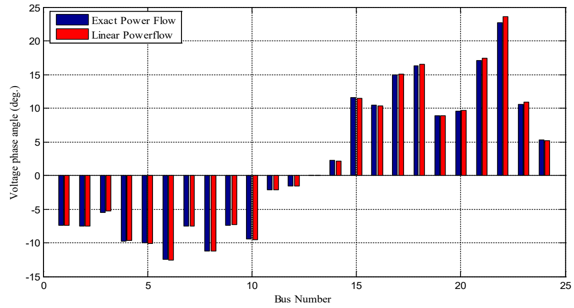

Fig. 6. Voltage phase angle validation through the approximated approach.

5.3 Results and Discussions

Considering the proposed framework, two overall system indices are deeply analyzed, including the annualized expected energy not supplied by EENS and PLC. Annualized indices are referred to as adequacy indices and are computed for a single load level. The annualized indices are obtained using the peak load level in this research.

Two main cases are devised herein. In the first case, these indices are evaluated, excluding the presence of TCPST. Considering the approach developed in Section 4, one TCPST device is in the system in the second case. Then, the values of the PLC and EENS indices are compared with those in the case without TCPST. To ensure statistical convergence, the MCS approach is executed for 2000 runs. As illustrated in Figs. 7 and 8, the EENS and PLC indices stabilize after approximately 1500 iterations, indicating sufficient sampling for reliable results. The moving average of these indices is monitored to confirm convergence before finalizing the set of scenarios used for optimization.

Both EENS and PLC are determined by adding up load curtailment statistics for all 2000 MCS runs. To assess sensitivity, we applied a ±10% change for specific values of FOR. Outcomes show less than 5% deviation in both indices, determining the robustness of the proposed framework in relation to practical uncertainties in component availability rates.

Table.3. Power flow calculations based on approximated and exact approach.

Bus Number	Nodal voltage magnitude (p.u.)			Nodal voltage phase angle (deg.)		
	Exact PF	Approximate PF	percent error	Exact PF	Approximate PF	percent error
1	1.0350	1.0350	0.0000	-7.4550	-7.3975	0.0077
2	1.0350	1.0350	0.0000	-7.5499	-7.4976	0.0069
3	0.9713	0.9724	0.1117	-5.4583	-5.2588	0.0365
4	0.9874	0.9872	0.0205	-9.7820	-9.6945	0.0089
5	1.0128	1.0113	0.1501	-10.0515	-10.1373	0.0085
6	1.0032	1.0012	0.1940	-12.4913	-12.6336	0.0114
7	1.0250	1.0250	0.0000	-7.5775	-7.4958	0.0108
8	0.9860	0.9853	0.0679	-11.2341	-11.1838	0.0045
9	0.9827	0.9821	0.0586	-7.3637	-7.2802	0.0113
10	1.0168	1.0151	0.1673	-9.4712	-9.5597	0.0093
11	0.9932	0.9924	0.0820	-2.1742	-2.1801	0.0027
12	1.0067	1.0058	0.0894	-1.5406	-1.5882	0.0309
13	1.0200	1.0200	0.0000	0	0	0
14	0.9800	0.9800	0.0000	2.2447	2.1674	0.0345
15	1.0140	1.0140	0.0000	11.5481	11.4718	0.0066
16	1.0170	1.0170	0.0000	10.4317	10.3542	0.0074
17	1.0386	1.0379	0.0597	14.9140	15.0434	0.0087
18	1.0500	1.0500	0.0000	16.2745	16.5010	0.0139
19	1.0232	1.0221	0.1123	8.8997	8.9010	0.0001
20	1.0385	1.0375	0.0988	9.5113	9.6715	0.0169
21	1.0500	1.0500	0.0000	17.0998	17.3945	0.0172
22	1.0500	1.0500	0.0000	22.7485	23.5846	0.0368
23	1.0500	1.0500	0.0000	10.5536	10.8738	0.0303
24	0.9821	0.9826	0.0537	5.2740	5.1135	0.0304
Total values	-	-	1.2660	-	-	0.3522

Similarly, results have been extracted for case 2, including the TCPST device allocation, by executing the proposed TCPST placement strategy in similar orders. The obtained results are reported in Fig. 9 for the PLCI. As seen, the line L_{27} has demonstrated the highest improvement in PLC index value. Hence, this line is selected as the optimal TCPST installation location intended to increase the power system reliability.

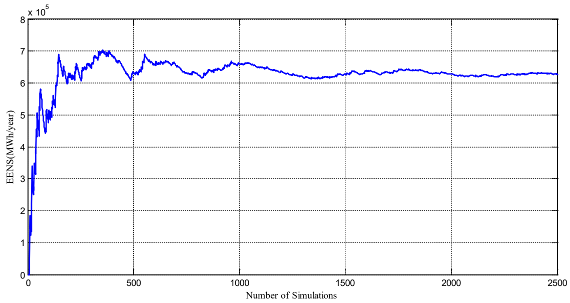

Fig. 7. EENS as a function of number of simulations.

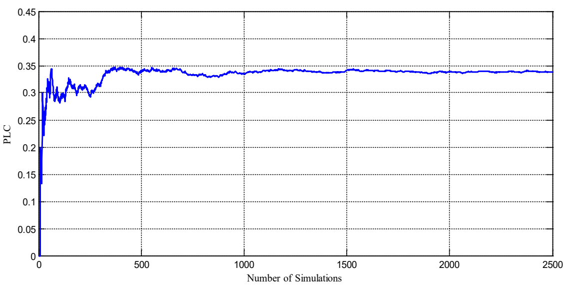

Fig. 8. PLC as a function of number of simulations.

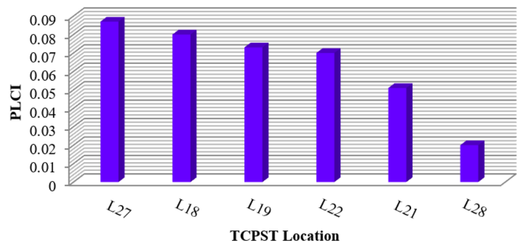

Fig. 9. PLCI after installing TCPST in different locations.

To evaluate the outperformance of the proposed placement strategy, all the candidate lines for TCPST placement which are determined based on the proposed approach are considered. Then, the PLC and EENS indices are calculated based on the MCS method. As shown from Fig. 10, installing TCPST device in L_{27}results in the best PLC index value. This observation reveals the outperformance of the proposed approach in optimal TCPST placement practice. Also, a fine attention on Fig. 11 demonstrates a consistency between the improvement in PLC index and EENS improvement. In other words, as the TCPST placement in line L_{27}has resulted in the minimum PLC, it also yields a minimum EENS value.

The obtained results for the two cases, with and without TCPST, are reported in Table.4. Regarding these results, installing a TCPST device at the Line L_{27}, enhanced the reliability indices, including PLC and EENS values, by 25.29 % and 17.14%, respectively.

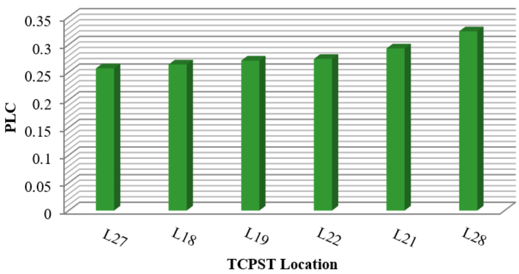

Fig. 10. PLC after installing TCPST in different locations.

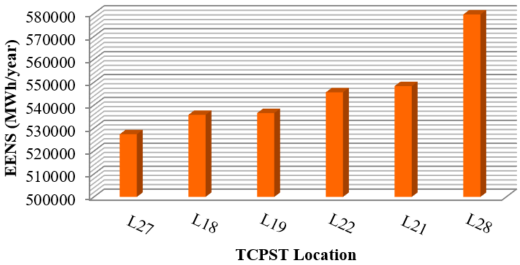

Fig. 11. EENS after installing TCPST in different locations.

Table 4. EENS and PLC in Case1 and Case 2.

Case studies	EENS (MWh/year)	PLC
Case 1	646640	0.344
Case 2	535805	0.257
Improvement	110835	0.087
Improvement (%)	17.14 %	

To evaluate the effectiveness of the proposed stochastic framework, a comparison was conducted with a deterministic TCPST placement approach in which average system parameters (without forced outage modeling) were considered. The deterministic model identifies candidate lines based solely on steady-state power flow conditions, neglecting probabilistic contingencies. Results indicate that while the deterministic approach yields lower computational time (as it avoids Monte Carlo sampling), it fails to capture load curtailment risk under component outages. In contrast, the proposed stochastic framework explicitly accounts for uncertainty through MCS and provides more reliable placement decisions reflected in reduced PLC and EENS values. Although the stochastic model requires higher computational effort, convergence analysis showed that approximately 1500–2000 simulation runs are sufficient to achieve stable reliability indices. Therefore, the additional computational burden is justified by the significant improvement in solution robustness and accuracy.

A sensitivity analysis was performed to assess the robustness of the PLCI index with respect to variations in FORs and the number of Monte Carlo simulations. FOR values were varied within ±10% of their nominal values. Results indicate that while absolute PLC values slightly change, the ranking of candidate lines based on PLCI remains consistent. This demonstrates that PLCI is structurally stable under moderate uncertainty variations. Additionally, increasing the number of simulations beyond 2000 did not significantly alter PLCI values, confirming convergence stability of the proposed index.

6 Conclusion

The probable occurrence of congestion in restructured power systems was thoroughly discussed as a new emerging challenge. As a remedial action, it was demonstrated that the placement of Thyristor-Controlled Phase-Shifting Transformers (TCPSTs) not only mitigates congestion in transmission lines but also enhances system reliability. The main objective of this study is to enhance the reliability of modern power systems through the optimal placement of TCPST under operational availability uncertainty. To achieve this goal, a novel probabilistic approach was developed for the TCPST placement problem, considering uncertainties associated with the availability of transmission lines and generating units. For effective scenario generation, the MCS method was employed. Given the large number of binary variables and the nonlinear nature of the developed model, approximation techniques were incorporated to improve the convergence behavior of the problem. A new index, named the PLCI, was introduced to rank the reliability improvement impact of TCPST placement on each line. For the tested system, both the EENS and the proposed PLCI indices were calculated using the MCS approach. Then, for each probable network topology, an optimal TCPST placement scheme was determined and ranked for the connecting transmission lines. The evaluation of the reliability indices for the optimally selected lines confirms the effectiveness of the proposed TCPST placement framework under operational and structural uncertainties. The reported improvements in EENS (17.14%) and PLC (25.29%) signify substantial gains in system reliability, reducing the likelihood of cascading failures and enhancing resilience during contingencies. These technical benefits suggest potential economic advantages through lower outage-related costs. Overall, this study contributes to this field by introducing a stochastic framework for optimizing TCPST under uncertainty. Its scientific value lies in the integration of probabilistic system states with reliability enhancement strategies. Practically, the results offer actionable insights for power system planners aiming to improve resilience in modern grids.

Although the proposed framework has been demonstrated on the modified IEEE RTS 24-bus system, its mathematical structure is not inherently limited to this scale. The optimization problem is formulated as a MILP model, where continuous variables associated with power flows, voltage magnitudes, phase angles, and load curtailment increase proportionally with the number of buses and transmission lines in each Monte Carlo scenario. Similarly, binary installation variables scale with the number of candidates TCPST locations. Since the linearized power flow formulation preserves convexity and avoids nonlinear coupling terms, the overall structure of the problem remains unchanged for larger transmission networks. Furthermore, the scenario-based Monte Carlo framework evaluates each system state independently, which enables parallel computation in large-scale applications. Therefore, the proposed methodology can be extended to real-world power systems with multiple TCPST installations without altering its fundamental formulation, while naturally incurring proportional growth in problem size. The present study focuses on reliability enhancement and does not explicitly incorporate investment and maintenance costs of TCPSTs. In practical planning applications, economic considerations such as capital cost, operational cost, and payback period should be integrated into a multi-objective optimization framework. Future research can extend the proposed methodology by combining PLCI with cost-benefit analysis to achieve economically optimal TCPST placement.

AI Use Statement

AI tools were used solely to assist with English-language writing and editorial refinement.

Disclosure of Potential Conflicts of Interest

The Authors declare that there is no conflict of interest

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Nomenclature	
Indices	
$i1$, i2	Indices for buses.
l	Indices of branches in transmission network.
s	Index for scenario.
Parameters and constants	
B	Susceptance of lines.
G	Conductance of lines.
B_{Sh}	Shunt charging susceptance of lines.
N_{br}	Number of branches in transmission network.
N_{PST}	Number of TCPSTs considered to be used in the system.
N_S	Number of simulations.
ω	Bus weighting factor related to load shedding policy.
Functions and Variables	
$P_{i1,i2}$	Active power flows from bus i1 to bus i2.
$Q_{i1,i2}$	Reactive power flows from bus i1 to bus i2.
P_{i1}^{D}	Active power demand at bus i1.
Q_{i1}^{D}	Reactive power demand at bus i1.
P_{i1}^{G}	Active power generation at bus i1.
Q_{i1}^{G}	Reactive power generation at bus i1.
$LC_{i1,s}^{A}$	Active load curtailment at bus i due to the contingency in s^{th}scenario.
$LC_{i1,s}^{R}$	Reactive load curtailment at bus i due to the contingency in s^{th}scenario.
$S_{l,s}$	Apparent power flow in line l
u	Binary installation variable
$\gamma_{i1,i2}$	Auxiliary variable approximating cosine term
V	Voltage magnitude.
δ	Phase angle.
$u_{i1,i2}$	Binary variable for TCPST status between busses i1 and i2.
$\varphi_{i1,i2}$	TCPST phase angle connected between i1 and i2.
$\gamma_{i1,i2}$	Piecewise linear approximation of $cos(\delta_i - \delta_j)$
$\underline{V_{i1}}$	Minimum voltage magnitude
$\overline{V_{i1}}$	Maximum voltage magnitude
$\underline{Q_{i1}^{G}}$	Minimum allowed active power injection
$\overline{Q_{i1}^{G}}$	Maximum allowed active power injection
$\underline{\varphi_{ij}}$	Minimum phase shift angle
$\overline{\varphi_{i1,i2}}$	Maximum phase shift angle