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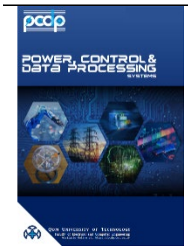

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An Optimal Location of the Measurement Devices in Distribution Network to Improve Estate Estimation Process Considering Reliability Issue

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ABSTRACT

In recent years, information security has become a significant issue in the energy sector. And the inability to control information can cause irreparable damage to the network. Therefore, we need measuring devices for better monitoring and control of the network, but due to financial limitations, it is not possible to install these devices in all nodes. In the state estimation process, the placement of measurement devices at optimal locations is of great importance. Therefore, by determining the best possible combination of measurement devices, including their type, location, and quantity, desirable results can be achieved. Therefore, this research presents the optimal placement of measuring devices with the aim of reducing the state estimation error. Also, the reliability of the measuring equipment and related costs are computed and optimized during the optimization procedure. The results of this research have been analyzed in two systems of 33 and 69 bus. The simulation results are compared with the conventional methods of the past and the results show the better performance of this method than the previous methods.

Keywords: State Estimation; Measurement Devices: Smart Network; Phasor Measurement Unit.

1 Introduction

The concept of smart grid introduces new issues in the performance of a distribution grid witch can improve its operation. The dispatching room of distribution grid must receive real-time data of the network to satisfy these issues effectively. In dispatching room of transmission grids, a state estimation module has been applied to attain the mentioned objectives, making it a natural approach for distribution systems as well [1]. Unlike to transmission grid, using of a classic state estimation is impossible because of complexities and unique specifications including radial topology of grid, high penetration of unbalanced loads, sparse measurements and having low ratio in X/R [2]. Convergence issues and poor state estimation can arise when applying transmission-based approaches to Electric Distribution Systems (EDS), necessitating additional robust techniques [3, 4]. The improve of a control actions supported by a state estimator is heavily affected by the accuracy of the estimation process [5]. Consequently, new methods and techniques have been explored to considerably increase the efficiency of state estimation module [6] .

Some research aim to increase the accuracy of the state estimation section [7, 8]. These studies emphasize the necessity of receiving data with acceptable accuracy and speed. In this field, synchrophasors have emerged as a desirable method to enhance custom control systems and information acquisition methods, such as Supervisory Control and Data Acquisition (SCADA) [9]. An integrated system including SCADA and PMUs was proposed in [10] to implement the state estimation process based on the Gauss-Newton method. This research employs a metric to allocate PMUs by considering three critical criteria: performance convergence and observability. Another research is addressed in [11] to effectively apply high-accuracy micro PMUs. The historical evolution of PMUs such as covering technological problems and main applications has been generally reviewed in [12]. Some authors present the importance of PMUs in increasing the vital performance of a distribution network such as control, monitoring and protection [13, 14]. Recent reviews highlight that actions across the entire network may not be necessary, as some CT and PT devices, along with information of grid configuration, can desirably increase observability in unobserved areas.

A metaheuristic algorithm has been proposed to provide SE with the minimum required measurements from PMUs, ensuring complete observability. In reference [15], a method is addressed to locate the measurement devices and define their types using automated meter readings, leveraging two state variables including the magnitude of current and phase angle. An optimal placement of PMUs is designed in [16] to improve the state estimation procedure for a three-phase EDS by optimizing an integer programming using greedy search technique.

In research [17], a communication availability based method is given to limit the actions required for monitoring power system situation such as voltage, stability and also to support the deployment of PMUs. A state estimation design is described in [18], which aims to increase stability through angle bias and address errors caused by issues in communication network. Authors in [19] introduce a nonlinear programming problem to enhance the possibility of measurements and observability by applying less number of PMUs and limited communication channels. A dynamic programming model is defined and optimized in [20] to reduce the number of required meters via incorporation the of random active power load variations during the state estimation procedure. Generally, the mentioned approaches do not considered the impact of grid changes on measurement redundancy, observability and also accuracy of state estimation procedure [21].

Due to the low levels of observability and automation in active distribution networks (ADN), deploying precise measurement devices to enhance network observability becomes inevitable. This paper develops an optimal framework for placing phasor measurement units (PMUs) considering the accuracy of the distribution system state estimation (DSSE) process [22].

This paper presents an algebraic technique to solve the PMU placement problem by focusing on full voltage state estimation and aiming to minimize the state estimation variance. The minimum number of required PMUs is obtained after optimizing the proposed model. Also, full voltage state observability is achieved in a network based on the number of buses with known nodal currents. The study also presents power system state estimation methods based on optimization over sensor networks. By minimizing a combined loss function while ensuring that state, disturbance, and measurement noise constraints are met, the best state estimations are iteratively computed. In this article, in addition to examining the proposed technique for the placement of measurement devices to achieve the minimum state estimation error, financial issues, costs incurred by the target networks, and reliability are also addressed. Efforts have been made to encompass all three aspects to obtain more realistic and practical results. Additionally, the paper proposes an optimal coordinated scheme to address observability and segmentation problems by determining the locations of PMUs and subsystems. The model also considers the impact of renewable energy sources on power system segmentation and the reliability value of power generation in the proposed framework.

The organization of this paper is as follows:

- Section 2 is dedicated to the theoretical foundations and research objectives
- Section 3 presents the simulation results obtained through the implementation of proposed method.
- Section 4 concludes the paper by summarizing the key findings and their implications.

2 Theoretical Foundations and Research Objectives

State estimation is considered one of the key components of network monitoring for electrical energy management. Therefore, the use of state estimation is inevitable. Due to the lack of complete and real-time information about the values of distribution network buses caused by the shortage of measurement devices, the management unit faces the challenge of unobservable network. By utilizing state estimation, power losses, voltage or power optimization, overload prevention of distribution lines, and more can be calculated. Consequently, network management using state estimation methods enhances capabilities such as control, monitoring, and load distribution in the distribution system, which ultimately improves power quality and increases consumer satisfaction. As explained so far, this study employs state estimation methods for more precise monitoring of the power network. Through this process, accurate information about the entire power network can be obtained. However, state estimation requires a set of initial data to perform its process. This data is obtained through measurement devices. These devices, when deployed in the network, collect the necessary information. Without such devices, state estimation cannot be performed. As a result, these measurement devices gather the data required for the state estimation process, which is then executed afterward.

In the following sections, the operational mechanism of the proposed method and the challenges associated with using measurement equipment are discussed. Since installing measurement devices at all network nodes is not feasible due to financial constraints, one of the most critical and challenging tasks in applying state estimation methods in distribution systems is the optimal placement of measurement devices in the distribution network. In this study, PMUs (Phasor Measurement Units) have been used as measurement devices. With the advancements in measurement technology, PMUs have been widely proposed for use in power distribution networks in recent research papers due to their numerous and beneficial capabilities. In other words, in modelling the optimal placement problem, the measured quantities play a significant role. These quantities can include active and reactive power, current, voltage, their angles, or a combination of these parameters. Considering the points discussed so far, the model presented in this study focuses on analysing state estimation errors. The proposed method in this research addresses not only state estimation but also the placement of measurement devices. Specifically, based on the number of PMUs determined for this model, the output identifies the buses where these devices should be installed, along with the corresponding state estimation error. Essentially, this method is designed to maximize efficiency while considering financial constraints and other factors important to stakeholders.

Equation (1) represents the objective function of the problem, where Z denotes the state estimation error [24]. The difference between the estimated values of voltage magnitude and angle and the corresponding values obtained from power flow analysis is considered the state estimation error in this problem.

$$Z = \sum_{k} PMU_k * \left|\frac{V_k - V_{pfk}}{V_{pfk}}\right| + \left|\frac{\theta_k - \theta_{pfk}}{\theta_{pfk}}\right| \qquad \text{Eq 1}$$

In this equation:

- PMUk is a binary variable indicating the presence or absence of a PMU at bus k,
- Vk represents the voltage magnitude at bus k,
- Vpf(k) denotes the voltage magnitude of bus kk obtained from power flow analysis,
- θ_k and θ_{pf_k} are the voltage angle at bus k and the voltage angle at bus k obtained from power flow analysis, respectively.

In this study, the results of power flow analysis under a specific network condition are used as the output of measurement devices due to the lack of real-world measurements.

$$\begin{aligned} P_{(k,m)} = (\mathrm{V}_k^2 * g_{(k,m)}) - (V_k * V_m * g_{(k,m)}) \\ * \mathrm{Cos}(\theta_k - \theta_m) - (V_k * V_m \\ * b_{k,m}) * \mathrm{Sin}(\theta_k - \theta_m) \end{aligned} \qquad \text{Eq 2}$$

$$\begin{aligned} Q_{(k,m)} = (-\mathrm{V}_k^2 * b_{(k,m)}) + (V_k * V_m * b_{(k,m)}) \\ * \mathrm{Cos}(\theta_k - \theta_m) - (V_k * V_m \\ * g_{(k,m)}) * \mathrm{Sin}(\theta_k - \theta_m) \end{aligned} \qquad \text{Eq 3}$$

Based on the article [23], from which Equations (2) and (3) are derived, these equations represent the active and reactive power flows through the transmission lines of the power network. These relationships depend on the voltage angle, voltage magnitude, and the conductance and susceptance matrices of the power network:

- $g_{k,m}$ is the conductance matrix at the fundamental frequency,
- $b_{k,m}$ is the susceptance matrix at the fundamental frequency,
- V with subscripts k,m represents the voltage magnitude at buses k,m,
- θ with subscripts k,m indicates the voltage angle at buses k,m and mm [23].

$$I_{(k,m)} = \sqrt{\begin{aligned} [(g^2{}_{(k,m)} + b^2{}_{(k,m)}) * (V^2{}_k + V^2{}_m \\ -2V_k * V_m) * \mathrm{Cos}(\theta_k - \theta_m)] \end{aligned}} \qquad \text{Eq 4}$$

The current flowing through the transmission lines of the network can also be determined using Equation (4). Similar to the active and reactive power equations, it depends on the values of susceptance, the conductance matrix, as well as the voltage magnitude and angle [23].

$$V_{\min} + ((1 - \delta) * V_{pf_k} - V_{\min} * PMU_k) \leq V_k \qquad \text{Eq 5}$$

$$V_k \leq V_{\max} + ((1 + \delta) * V_{pf_k} - V_{\max} * PMU_k) \qquad \text{Eq 6}$$

The minimum and maximum voltage values of network buses are represented in Equations (5) and (6). These equations depend on the presence or absence of a PMU at the corresponding bus and are also influenced by the allowable minimum and maximum voltage limits, as well as the measurement device error. In these equations, $V_{pf(k)}$ represents the voltage magnitude of bus k obtained from power flow analysis, and PMU_k is a binary variable indicating the presence or absence of a PMU at bus k. If no measurement device is present at a bus in this proposed method (PMUk=0), it is sufficient for the estimated voltage to remain within the allowable maximum and minimum voltage range. Conversely, if a measurement device is present at a bus (PMUk=1), the constraints in Equations (5) and (6) set the voltage magnitude of the bus to equal the actual measured value (from power flow analysis), plus an error margin that accounts for the measurement device's error [25].

$$R_i(t) = A_i.e^{-\alpha_i t} \qquad \text{Eq 7}$$

Equation (7) represents the reliability of each PMU, and this value is obtained using the following formula [30]. First, the reliability of each measurement device is calculated, which can vary depending on the model and brand. Once this value is obtained, the unreliability is determined. Finally, the overall network reliability is calculated. Naturally, increasing the number of measurement devices leads to an increase in the overall reliability of the network. The test networks used in this study are the IEEE 33-bus and 69-bus systems. The process begins with the 33-bus network, where the proposed method is fully implemented. Afterward, the simulation results are analysed and evaluated. These results are obtained for different numbers of measurement devices and compared. The purpose of this comparison is to demonstrate the performance of the proposed method. It is worth noting that the same procedure is applied to the 69-bus network, and the results are compared accordingly. The procedure begins by inputting a set of system data, which include the specifications of the respective 33-bus or 69-bus network. Next, the mentioned formulas and the objective function are implemented to analyse the obtained results. Through an iterative process, the state estimation error is calculated for the defined number of PMUs. It is evident that the minimum state estimation error for the given network corresponds to the optimal number of PMUs, which is the objective of the problem. This process is repeated until the minimum state estimation error is achieved. During this iterative process, the coding ensures that all candidates are evaluated. Finally, the desired objective function is obtained. The bus with the lowest state estimation error is selected as the optimal location for PMU installation. This approach not only minimizes the state estimation error but also determines the best PMU installation locations. This process can be repeated for different numbers of PMUs, and the results can be obtained separately. The same procedure is applied to both the 33-bus and 69-bus networks. Generally, the flowchart of the proposed method is shown in Figure 1.

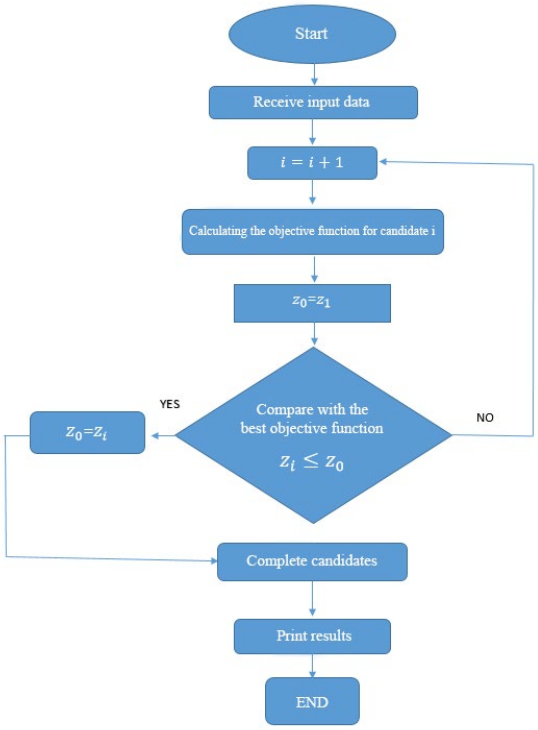

Figure 1: Flowchart of the proposed method

3 Simulation Results

In this section, the simulation results of this study are analysed. Initially, the effect of the number of measurement devices on the objective function is examined. Subsequently, the simulation results under identical conditions are compared with traditional methods, specifically the WLS (Weighted Least Squares) method, which is used as a benchmark in this paper. It should be noted that the simulations were carried out using the GAMS software, and the obtained results and graphs were derived from this software. First, the impact of increasing the number of PMUs on state estimation error was studied. The initial experiment was conducted on the IEEE 33-bus network. As expected, increasing the number of measurement devices reduced the state estimation error. This trend continued up to 11 measurement devices for the 33-bus network. Beyond this number, there was no significant reduction in state estimation error, and further increases were not pursued. Additionally, due to the high cost of these devices, unnecessary installations would impose a substantial financial burden on the distribution company. Thus, optimizing the number and placement of PMUs is crucial to balancing cost and performance.

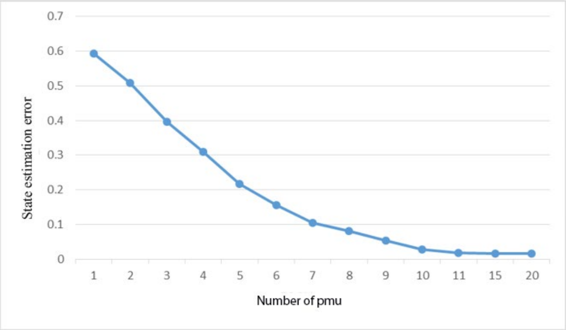

Figure 2: The impact of increasing PMUs on state estimation error

The same experiment was conducted on the IEEE 69-bus network. As expected, increasing the number of PMUs resulted in a reduction in state estimation error. For the 69-bus network, the number of PMUs was increased up to 24, and the state estimation error was calculated. Beyond this number, no significant reduction in state estimation error was observed. Therefore, the number of PMUs was not increased further.

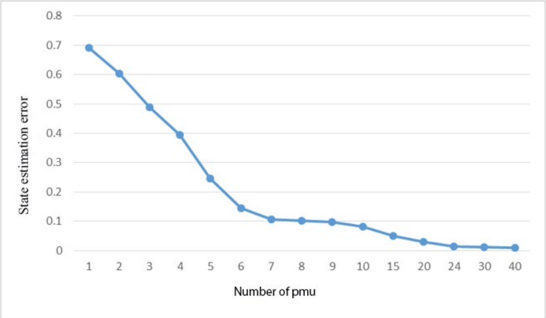

Figure 3: The impact of different numbers of PMUs on state estimation error

Now, using the obtained results, the optimal locations for installing measurement devices are analysed. Selecting these installation locations plays a crucial role in reducing the state estimation error. While the objective function of the study is to minimize the state estimation error, this cannot be achieved without determining the optimal locations for the measurement devices. Therefore, alongside evaluating the objective function for the 33-bus network, the installation locations of the measurement devices must also be determined. This process will be carried out using the proposed formulation, and the results will be visualized in the simulation environment. The table 1 presents the results of placing different numbers of PMUs in optimal locations corresponding to the percentage error for each case.

Table 1: Error magnitude in relation to different numbers of PMUs.

number of PMUs	Installation location	State estimation error
1	1	0.594
5	1,5,7,15,1,21	0.217
11	1,2,3,5,9,13,16,19,23,27,29,31	0.018

Based on the obtained data from the proposed method, the estimated voltage magnitude and angle are analysed. The aim of this analysis is to show the accurate results obtained from state estimation for better representation of the outcomes. Initially, the voltage magnitude is analysed using three, five, and eleven measurement devices, and the results are compared with the reference values (actual measurements). The comparison results are shown in Figure 4.As observed, increasing the number of measurement devices (PMUs) brings the estimated voltage magnitude closer to the reference values obtained from actual measurements. This comparison suggests that by placing the measurement devices in optimal locations, the results can approach the reference values more closely.

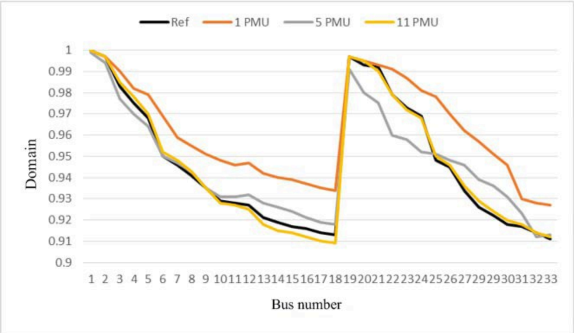

Figure 4: The impact of different numbers of PMUs on voltage magnitude value

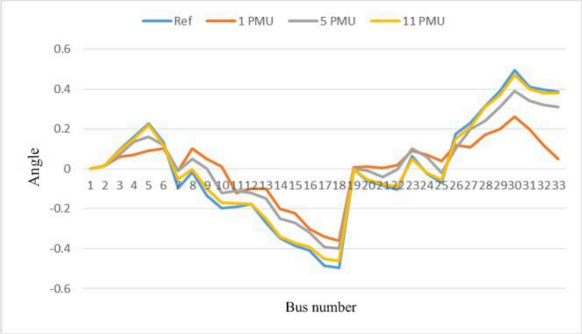

Figure 5: The impact of different numbers of PMUs on voltage angle value

From Figure 5, it can be concluded that increasing the number of measurement devices has a positive impact on the voltage angle, similar to its effect on the voltage magnitude. With an increase in the number of PMUs, the voltage angle can be brought closer to the reference value (the actual measured value). This suggests that increasing the number of PMUs in the state estimation process is beneficial and yields desirable results. After analysing the 33-bus network, we now turn to the analysis of the IEEE 69-bus network. Initially, different numbers of measurement devices are tested on this network, and the state estimation error is obtained corresponding to these numbers of devices. As the number of PMUs increases, the state estimation error decreases. The number of these devices is increased until the error no longer shows significant changes. As observed, this increase continued up to 24 devices, and beyond this number, no significant reduction in state estimation error was observed. After examining the effect of increasing the number of PMUs on reducing the state estimation error, the next step is to analyse the error by finding the optimal installation locations for the measurement devices. As mentioned earlier, identifying the best installation locations for the measurement devices has a significant impact on reducing the state estimation error. This information is also very useful for electric companies, as it helps them identify which buses in a 69-bus network should be equipped with PMUs to achieve the best state estimation. Table (2) displays the state estimation error corresponding to different numbers of PMUs installed at the optimal locations for the measurement devices.

Table 2: State estimation error magnitude in relation to different numbers of PMUs.

Number of PMUs	Installation location	State estimation error
1	1	0.691
5	1,4,5,32,46	0.211
10	1,4,7,24,33,39,43,46,56,69	0.081
15	1,4,7,9,13,23,31,34,43,46,52,57,61,65	0.049
24	1,2,4,7,13,14,19,21,24,26,29,31,34,37 ,39,42,46,49,54,58,61,64,67,69	0.014

As shown in Table 3, the optimal installation positions for one to twenty-four PMUs are specified, and it demonstrates that as the number of these measurement devices increases, the state estimation error shows a decreasing trend. It is possible to further increase the number of measurement devices, but since the decrease beyond this point is not significant, it is not financially justifiable to increase the number any further. This is because each measurement device has a relatively high cost, and electric companies are limited by their financial resources.

After analysing the state estimation error based on the optimal installation locations of the measurement devices, the next step is to examine the voltage magnitude and angle in the 69-bus network. First, the voltage magnitude is analysed based on different numbers of measurement devices. In this process, as the number of PMUs increases, the results are compared with the actual measured values. As shown in Figure 6, the voltage magnitude values are obtained for one, five, ten, fifteen, twenty, and twenty-four devices.

As observed, with the increase in the number of measurement devices, the obtained voltage magnitude approaches the reference value (actual measurement). This indicates the positive impact of increasing the number of PMUs in the state estimation process, as better results are obtained with more devices. The same experiment was conducted for the voltage angle as well. Just as with the voltage magnitude, the voltage angle was analysed in the same way. The results of this comparison are shown in Figure 7.

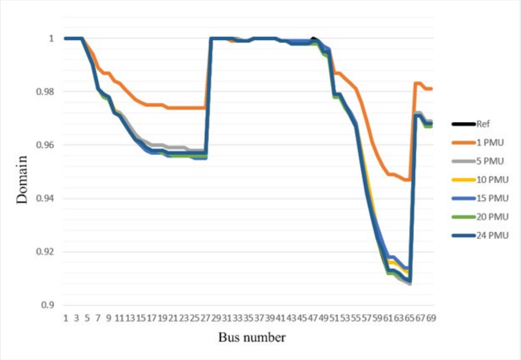

Figure 6: The impact of different numbers of PMUs on voltage magnitude

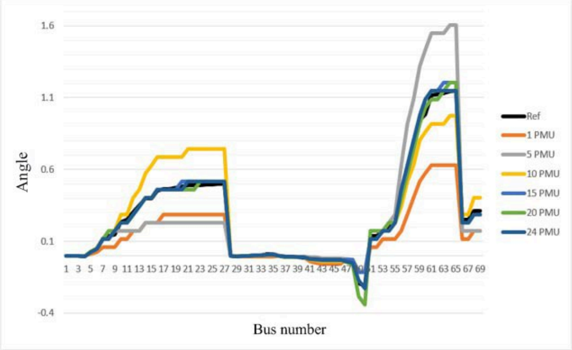

Figure 7: The impact of different numbers of PMUs on voltage angle

As shown in the figure, for different numbers of PMUs, the voltage angle in the system (a 69-bus system) was determined. It is observed that with the increase in the number of PMUs, the voltage angle approaches the reference value (actual measurement). This result indicates that by increasing the number of measurement devices and placing them in the appropriate locations, optimal state estimation can be achieved. It is worth noting that the load flow values were used in the actual measurements.

Up to this point, we have observed that increasing the number of PMUs can not only reduce state estimation errors but also bring the obtained values of voltage magnitude and angle closer to the actual measured values. To further illustrate the results obtained from the proposed method, a comparison is made with a conventional method. To better understand the applicability and results of the proposed method, the results are compared with the traditional Weighted Least Squares (WLS) method. For a fair comparison, the results are examined with an equal number of PMUs, specifically with 3 PMUs. The comparison of the voltage magnitude with 3 PMUs is shown in Figure 8, and the results indicate an improvement in the performance of the proposed method over the WLS method. The voltage magnitude obtained using the proposed method is much closer to the reference value than that of the traditional Weighted Least Squares method.

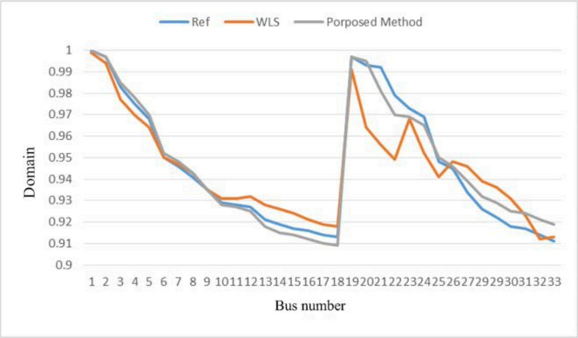

Figure 8: Comparison of the voltage magnitude between the proposed method and the WLS method for three PMUs.

Similarly, the voltage angle values obtained using three PMUs are compared for both methods in Figure 9. Just like the voltage magnitude, the proposed method provides values closer to the reference value, indicating that the proposed method outperforms the conventional method in terms of accuracy. This further emphasizes the effectiveness of the proposed approach in achieving more precise state estimation.

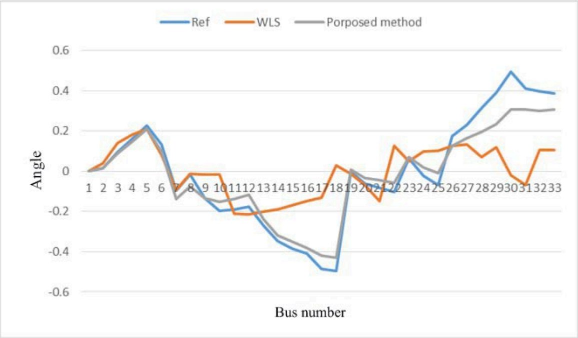

Figure 9: Comparison of the voltage angle between the proposed method and the WLS method for three PMUs.

From this point onward, the proposed method continues to be closer to the reference value, but due to the convergence of the two methods at higher PMU counts, the results in the figures become indistinguishable and are very close to each other. However, the proposed method still provides better state estimation with voltage magnitude and angle values closer to the reference. Overall, it can be concluded that with the proposed method, better results can be achieved by increasing the number of PMUs compared to the conventional weighted least squares (WLS) method. This trend of decreasing state estimation error in the proposed method, as shown in the figures, is much faster than in the WLS method. Then, the reliability analysis was performed for a 33-bus network. As evident, with the increase in the number of measuring devices, reliability also improves. It is important to note that reliability varies depending on the model and brand of each measuring device. In this study, the reliability was analysed for a specific device model, but this value can differ slightly across different brands. However, these differences are generally not significant, though they may lead to somewhat different results. As mentioned earlier, the reliability increases with the number of measuring devices. However, this increase continues only up to a certain point and approaches a value of 1 (maximum reliability).

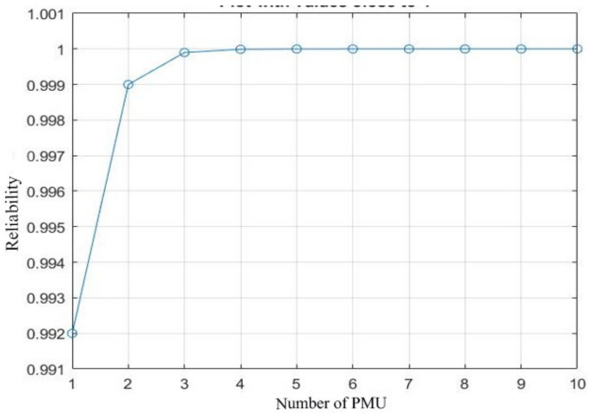

Figure 10: Reliability analysis in a 33-bus power system.

As shown in Figure 10, with the increase in the number of PMUs beyond four, the reliability approaches its maximum value, and after that, no significant change is observed in the reliability. It can be concluded that by increasing the number of PMUs in a 33-bus network, reliability increases up to a certain point, after which it reaches its maximum and no longer changes.

This thesis also examines the costs associated with installing PMUs in the 33-bus and 69-bus networks. The cost varies depending on the number of PMUs used in the network. Additionally, this cost depends on other factors such as the type and model of the PMU. The cost includes procurement, installation, and commissioning of the PMUs in the network. Therefore, the cost analysis for both networks is presented in Tables (3) and (4).

Table 3: Cost representation for a 33-bus power system.

Test network	Number of PMU	Cost
33 bus	1	$40000
33 bus	5	$200000
33 bus	11	$440000

Table 4: Cost representation for a 69-bus power system.

Test network	Number of PMU	Cost
69 bus	1	$40000
69 bus	5	$200000
69 bus	10	$400000
69 bus	15	$600000
60 bus	20	$800000
69 bus	24	$960000

4 Conclusion

In this research, given the high importance of state estimation accuracy in all parts of the power distribution network, improving state estimation accuracy is the primary priority, while other factors are considered as constraints of the problem. Most previous works that used state estimation methods for power distribution networks did not provide acceptable accuracy and convergence. One of the main challenges in implementing the process of sensor placement is the development and advancement of state estimation algorithms tailored to the real-world conditions of distribution networks, compared to transmission networks. This research addresses this issue by providing practical methods to adapt the state estimation algorithm to the specific characteristics of distribution networks, which is one of the main objectives of the thesis. The proposed model in this research investigates state estimation errors. In addition to calculating state estimation, the proposed method also considers the placement of sensors. Based on the number of PMUs determined, the output specifies the bus numbers where the sensor devices should be installed, along with the state estimation errors. This method can achieve maximum efficiency by considering limitations in budget and other elements important to the client. The results obtained were simulated for two different systems, 33-bus and 69-bus, and the effect of the number of measurement devices in each of the mentioned systems was analysed. Based on the simulation results, the obtained results are compared with the conventional WLS method.

Finally, according to the simulation results, it was found that increasing the number of measurement devices reduces the percentage of state estimation errors. The proposed method in this research achieves a more noticeable reduction in state estimation errors compared to the conventional WLS method. Moreover, in the proposed method, the percentage of state estimation errors decreases much more rapidly as the number of measurement devices increases. Therefore, it can be concluded that the performance of the proposed method in this research, compared to the WLS method, is better in reducing state estimation errors. Additionally, reliability was also examined in this research, and it was concluded that increasing the number of measurement devices can enhance the network's reliability. Finally, the costs for the 33-bus and 69-bus networks were analysed and determined.

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